

Страница 69 — Модулни уравнения

Задача 1: $|2.x - 1| + 3.|2.x - 1| = 12$

$$4.|2.x - 1| = 12 \Rightarrow |2.x - 1| = 3$$

$$1) 2.x - 1 = 3 \Rightarrow 2.x = 4 \Rightarrow x_1 = 2$$

$$2) 2.x - 1 = -3 \Rightarrow 2.x = -2 \Rightarrow x_2 = -1$$

Задача 2: $7.|x + 3| - 5.|x + 3| = 6$

$$2.|x + 3| = 6 \Rightarrow |x + 3| = 3$$

$$1) x + 3 = 3 \Rightarrow x_1 = 0$$

$$2) x + 3 = -3 \Rightarrow x_2 = -6$$

Задача 3: $|x + 3| - |5.x + 15| = -8$

$$|x + 3| - 5.|x + 3| = -8 \Rightarrow -4.|x + 3| = -8 \Rightarrow |x + 3| = 2$$

$$1) x + 3 = 2 \Rightarrow x_1 = -1$$

$$2) x + 3 = -2 \Rightarrow x_2 = -5$$

Задача 4: $|6.x - 2| - |9.x - 3| = -5$

$$2.|3.x - 1| - 3.|3.x - 1| = -5 \Rightarrow -|3.x - 1| = -5 \Rightarrow |3.x - 1| = 5$$

$$1) 3.x - 1 = 5 \Rightarrow x_1 = 2$$

$$2) 3.x - 1 = -5 \Rightarrow x_2 = -4/3$$

Задача 5: $|x^2 - 2.x - 4| = 4$

$$1) x^2 - 2.x - 8 = 0 \Rightarrow x = \frac{-(-2) \pm \sqrt{((-2)^2 - 4 \cdot 1 \cdot (-8))}}{2 \cdot 1} = \frac{2 \pm 6}{2} \Rightarrow x_1 = 4, x_2 = -2$$

$$2) x^2 - 2.x = 0 \Rightarrow x \cdot (x - 2) = 0 \Rightarrow x_3 = 0, x_4 = 2$$

Задача 6: $|x^2 + 2.x - 4| = 4$

$$1) x^2 + 2.x - 8 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{(2^2 - 4 \cdot 1 \cdot (-8))}}{2 \cdot 1} = \frac{-2 \pm 6}{2} \Rightarrow x_1 = 2, x_2 = -4$$

$$2) x^2 + 2.x = 0 \Rightarrow x_3 = 0, x_4 = -2$$

Задача 7: $|x^2 - 2.x - 1| = 2$

$$1) x^2 - 2.x - 3 = 0 \Rightarrow x = \frac{2 \pm \sqrt{(4 - 4 \cdot 1 \cdot (-3))}}{2} = \frac{2 \pm 4}{2} \Rightarrow x_1 = 3, x_2 = -1$$

$$2) x^2 - 2.x + 1 = 0 \Rightarrow (x - 1)^2 = 0 \Rightarrow x_3 = 1$$

Задача 8: $|x^2 + 5.x| = 6$

$$1) x^2 + 5x - 6 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{(25 - 4 \cdot 1 \cdot (-6))}}{2} \Rightarrow x_1 = 1, x_2 = -6$$

$$2) x^2 + 5x + 6 = 0 \Rightarrow x = \frac{-5 \pm \sqrt{(25 - 24)}}{2} \Rightarrow x_3 = -2, x_4 = -3$$

Задача 9: $|x^2 + 4x| = 3$

$$1) x^2 + 4x - 3 = 0 \Rightarrow x = \frac{-4 \pm \sqrt{(16 - 4 \cdot 1 \cdot (-3))}}{2} = -2 \pm \sqrt{7}$$

$$2) x^2 + 4x + 3 = 0 \Rightarrow x = \frac{-4 \pm 2}{2} \Rightarrow x_3 = -1, x_4 = -3$$

Задача 10: $|3x^2 - 5x + 1| = 1$

$$1) 3x^2 - 5x = 0 \Rightarrow x_1 = 0, x_2 = 5/3$$

$$2) 3x^2 - 5x + 2 = 0 \Rightarrow x = \frac{5 \pm \sqrt{(25 - 24)}}{6} \Rightarrow x_3 = 1, x_4 = 2/3$$

Задача 11: $|x^2 + 5x + 4| = 4$

$$1) x^2 + 5x = 0 \Rightarrow x_1 = 0, x_2 = -5$$

$$2) x^2 + 5x + 8 = 0 \Rightarrow D = 25 - 32 = -7 \Rightarrow \text{Няма реални корени.}$$

Задача 12: $|x^2 - 4x + 1,5| = 1,5$

$$1) x^2 - 4x = 0 \Rightarrow x_1 = 0, x_2 = 4$$

$$2) x^2 - 4x + 3 = 0 \Rightarrow x = \frac{4 \pm 2}{2} \Rightarrow x_3 = 3, x_4 = 1$$

Задача 13: $|x^2 - 7x + 3| = 3$

$$1) x^2 - 7x = 0 \Rightarrow x_1 = 0, x_2 = 7$$

$$2) x^2 - 7x + 6 = 0 \Rightarrow x = \frac{7 \pm 5}{2} \Rightarrow x_3 = 6, x_4 = 1$$

Задача 14: $|x^2 + 2x + 9| = 1$

$$1) x^2 + 2x + 8 = 0 \Rightarrow D = 4 - 32 < 0$$

$$2) x^2 + 2x + 10 = 0 \Rightarrow D = 4 - 40 < 0 \Rightarrow \text{Няма реални корени.}$$

Задача 15: $|x^2 - 3x + 2| = 0$

$$x^2 - 3x + 2 = 0 \Rightarrow x = \frac{3 \pm \sqrt{(9 - 8)}}{2} \Rightarrow x_1 = 2, x_2 = 1$$

Задача 16: $|2x^2 - 3x - 2| = -1$

Няма решение (модулът не може да е отрицателен).

Задача 17: $|(x + 3)^2 - 2(x + 1)| = 4$

$$x^2 + 4x + 7 = 4 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow x = \frac{-4 \pm 2}{2} \Rightarrow x_1 = -1, x_2 = -3$$

Задача 18: $|(x - 2)^2 - 2 \cdot (x - 1)| = 1$

$$x^2 - 6x + 6 = 1 \Rightarrow x^2 - 6x + 5 = 0 \Rightarrow x = \frac{6 \pm 4}{2} \Rightarrow x_1 = 5, x_2 = 1$$

$$x^2 - 6x + 6 = -1 \Rightarrow x^2 - 6x + 7 = 0 \Rightarrow x = \frac{6 \pm \sqrt{8}}{2} = 3 \pm \sqrt{2}$$

Задачи 19 - 24:

19) $x = \pm\sqrt{3}, x = \pm 1$

20) $x = \pm\sqrt{3}/2, x = \pm 1/2$

21) $x = 0, x = \pm\sqrt{5}, x = \pm 2, x = \pm 1$

22) $x = 0, x = \pm 2, x = \pm\sqrt{3}, x = \pm 1$

23) $x = 0, x = \pm\sqrt{7}, x = \pm\sqrt{6}, x = \pm 1$

24) $x = \pm\sqrt{2}, x = 0$

Страница 75 — Показательни уравнения

Задача 5

а) $x^2 + 3x - 4 = 0 \Rightarrow x = \frac{-3 \pm \sqrt{(9 - 4 \cdot 1 \cdot (-4))}}{2 \cdot 1} \Rightarrow x = 1, x = -4$

б) $x^2 + 2x - 3 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{(4 - 4 \cdot 1 \cdot (-3))}}{2 \cdot 1} \Rightarrow x = 1, x = -3$

в) $x = 1, x = -3$

г) $x^2 + x - 2 = 0 \Rightarrow x = 1, x = -2$

Задача 6

а) $7 \cdot 2^x = 28 \Rightarrow 2^x = 4 \Rightarrow x = 2$

б) $8 \cdot 3^x = 72 \Rightarrow 3^x = 9 \Rightarrow x = 2$

в) $5 \cdot 7^x = 245 \Rightarrow 7^x = 49 \Rightarrow x = 2$

г) $18 \cdot 5^x = 450 \Rightarrow 5^x = 25 \Rightarrow x = 2$

Задача 7

а) $3 \cdot 2^x = 48 \Rightarrow x = 4$ | б) $5 \cdot 3^x = 135 \Rightarrow x = 3$ | в) $43 \cdot 7^x = 301 \Rightarrow x = 1$ | г) $50 \cdot 5^x = 250 \Rightarrow x = 1$

Задача 8

а) $x^2 + 5x - 6 = 0 \Rightarrow x = \frac{-5 \pm 7}{2} \Rightarrow x = 1, x = -6$

б) $x^2 + 3x = 0 \Rightarrow x = 0, x = -3$

в) $x^2 + 3x - 4 = 0 \Rightarrow x = 1, x = -4$

г) $x^2 + 4x - 5 = 0 \Rightarrow x = 1, x = -5$

Задача 9

а) $5x - 2 = -3x + 6 \Rightarrow x = 1$ | б) $4x = -12 \Rightarrow x = -3$ | в) $5x = -10 \Rightarrow x = -2$ | г) $7x = 0 \Rightarrow x = 0$

Задача 10

а) $x^2 + 2x - 3 = 0 \Rightarrow x = 1, x = -3$ | б) $x^2 + 4x + 3 = 0 \Rightarrow x = -1, x = -3$
 в) $x^2 + 5x + 4 = 0 \Rightarrow x = -1, x = -4$ | г) $x^2 + 2x - 3 = 0 \Rightarrow x = 1, x = -3$

Задача 11

а) $x = -2$ | б) $x = -1$ | в) $x = -14$ | г) $x = -8$

Задача 12

а) $x = \log_{3/2} 2$ | б) $x = \log_{5/3} (1/9)$ | в) $x = \log_{7/4} (1/8)$ | г) $x = \log_{7/5} (1/3125)$

Страница 79 — Показателни уравнения с полагане

Задачи 1 - 5 ($y = a^x$)

1) $y^2 - 10y + 16 = 0 \Rightarrow y = \frac{10 \pm \sqrt{(100 - 64)}}{2} = \frac{10 \pm 6}{2} \Rightarrow y = 8, y = 2 \Rightarrow x = 3, x = 1$
 2) $y^2 - 10y + 9 = 0 \Rightarrow y = 9, y = 1 \Rightarrow x = 2, x = 0$
 3) $y^2 - 26y + 25 = 0 \Rightarrow y = 25, y = 1 \Rightarrow x = 2, x = 0$
 4) $y^2 - 5y + 6 = 0 \Rightarrow y = 3, y = 2 \Rightarrow x = 1, x = \log_3 2$
 5) $y^2 - 6y + 5 = 0 \Rightarrow y = 5, y = 1 \Rightarrow x = \log_7 5, x = 0$

Задачи 6 - 15

6) $y^2 + 2y - 80 = 0 \Rightarrow y = 8 \Rightarrow x = 3$
 7) $y^2 - 2y - 8 = 0 \Rightarrow y = 4 \Rightarrow x = \log_3 4$
 8) $y^2 - 120y - 625 = 0 \Rightarrow y = \frac{120 \pm 130}{2} \Rightarrow y = 125 \Rightarrow x = 3$
 9) $y^2 - 8y - 9 = 0 \Rightarrow y = 9 \Rightarrow x = 2$
 10) $27y^2 - 6y - 1 = 0 \Rightarrow y = 1/3 \Rightarrow x = -1$
 11) $y^2 - 20y + 64 = 0 \Rightarrow y = 16, y = 4 \Rightarrow x = 2, x = 1$
 12) $t = x^2 - x \Rightarrow y^2 - 2y - 8 = 0 \Rightarrow y = 4 \Rightarrow t = 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, x = -1$
 13) $y^2 - 12y + 27 = 0 \Rightarrow y = 9 \Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}; y = 3 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$
 14) $y^2 - 10y + 16 = 0 \Rightarrow 2^{\sqrt{x-2}} = 8, 2 \Rightarrow x = 11, x = 3$
 15) $y^2 - 12y + 27 = 0 \Rightarrow 3^{\sqrt{3}x} = 9, 3 \Rightarrow x = 4/3, x = 1/3$

Задачи 16 - 20 (Хомогенни)

16) $x = 0$ | 17) $x = 0$ | 18) $x = 0$ | 19) $x = 0, x = 1$ | 20) $x = 2$

Страница 83 — Разнородни задачи

1) $x = 3$ | 2) $x = 2, x = 0$ | 3) $x = 2, x = 0$ | 4) $x = 2$ | 5) $x = 0$ | 6) $x = 0$
 7) $2y^2 - 3y - 20 = 0 \Rightarrow y = \frac{3 \pm 13}{4} \Rightarrow y = 4 \Rightarrow 4^x = 4 \Rightarrow x = 1$
 8) $3y^2 - 8y - 35 = 0 \Rightarrow y = \frac{8 \pm 22}{6} \Rightarrow y = 5 \Rightarrow 5^x = 5 \Rightarrow x = 1$
 9) $x = 0, x = 3$ | 10) $x = 1, x = 0$ | 11) $x = -1, x = 1$ | 12) $x = 2, x = 3$